QMSs: Decoherence and empirical estimates
Genova, 26-28 June 2013

Three new principles in non–equilibrium statistical mechanics and their mutual relationships

L. Accardi

Both in classical and quantum statistical mechanics several equivalent characterizations of equilibrium states are known, e.g.:
– the Boltzmann-Gibbs prescription,
– the KMS condition,
– the detailed balance condition.
The 'essential' equivalence of these three conditions was recognized in several decades of investigations.

Until recently however no non-equilibrium analogue of these conditions were known. Even worse: no principle was known, general enough to include large classes of interesting non equilibrium states, but not so vague to put together states which have hardly anything in common (as it turned out to be the case for some more or less recent proposals).

In the past 12 years the stochastic limit of quantum theory suggested the following natural generalizations of the above notions:
- the **nonlinear** Boltzmann-Gibbs prescription
- the **local** KMS condition
- the **dynamical** detailed balance condition.

The fact that these three conditions are equivalent under rather general assumptions is non trivial.

The main goals of the present talk are:
– To describe these new notions and the ideas that led to their introduction.
– To illustrate the conditions of their mutual equivalence.
– To discuss the new class of quantum Markov semi-groups arising in connection with these non-equilibrium phenomena and the very special mathematical properties which distinguish them from the semi-groups arising in equilibrium situations.

If time allows we will also describe some new physical effects emerging in these states, for example:
– a nonlinear version of the Einstein coefficients in standard radiation theory,
– the phenomenon of *purity generating quantum Markov semi-groups*,
– applications to the *control through decoherence* program in the theory of quantum control,
... .

This line of research was initiated in 2002 in a joint paper with K.Imafuku (see (1)).
The more recent results, which I hope I will be able to discuss in the present exposition are contained in a joint paper with F. Fagnola, R. Quezada (see (2)).

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Full version in preparation for: Infinite Dimensional Analysis, Quantum Probability and Related Topics

Quantum dynamical processes: Hilbert space and phase space structures
B. Baumgartner

Abstract

Quantum dynamical semigroups acting in systems with finite dimensional Hilbert space show in their long time asymptotics Decay, Dissipation, Dephasing and Decoherence. These typical non equilibrium quantum mechanical phenomena are related, in a precise way, to decompositions of the Hilbert space and representations of density matrices in block form. Positivity of the mappings and formation of a Markovian semigroup are the key ingredients, the basis for the analysis. Continuity and complete positivity are needed in specifying some details.

Quantum dynamical semigroups in infinite dimensions
B. Baumgartner

Abstract

Only for norm continuous semigroups of completely positive maps the generators can be given well defined standard forms, established by Gorini, Kossakowski and Sudarshan and by Lindblad. For strongly continupus semigroups this task has not yet been completely fulfilled. I present the achievements of E. B. Davies, of A. S. Holevo, and my own results, aiming at establishing a generalized standard form for the generators.

MUness and SICness of covariant phase space observables
C. Carmeli

Abstract

We show how, in any prime power dimension, covariant phase space observables give a complete family of mutually unbiased POVMs that are smearings of mutually unbiased bases (MUBs). We give an operational characterization of symmetric informationally complete (SIC) phase space observables.

Effect of cavity decay on Rabi flopping and phase distribution in cavity QED
P.K.Das

Abstract

The population inversion and phase distribution of states in a cavity driven by classical field are analyzed. A two-level atom is sent through the cavity and driven by a classical field. Also the time evolution of the system includ- ing decay is obtained. The graph of the Rabi oscillations for this system is shown to be changed from the system without decay. The phase probability distribution is seen to be affected due to the presence of decay in the cavity.

Quantum observables identifying all states with low rank
T. Heinosaari

Abstract

The basic task in quantum tomography is to identify an unknown quantum state from measurement outcome statistics. However, in a realistic situation the state is not completely unknown but there is a premise on it, hence limiting the set of possible states. This additional aspect leads naturally to the question of verifying the premise with respect to some looser premise or no premise at all. We investigate quantum tomography under these two supplemental features, concentrating to the cases when the initial premise and the verification premise are statements about the rank of the unknown state. We characterize the structure of quantum observables (POVMs) that are capable of fulfilling this type of identification tasks.

Open system identification: asymptotic normality and Fisher information
M. Guta

Abstract

We consider the problem of estimating a dynamical parameter of an open system coupled with an environment, in the input-output formalism. For ergodic master evolutions we compute the asymptotic quantum Fisher information (per time unit) of the output, and show that the latter is locally asymptotically normal, i.e. it can be approximated by a coherent state in a statistical sense. We further show that the integrated statistics of simple measurements (e.g. counting and homodyne) are asymptotically normal and compute the associated classical Fisher information. The results are applied to computing Fisher informations for different statistics in the atom maser.

Density theorems in quantum harmonic analysis
J. Kiukas

Abstract

A covariant phase space observable is uniquely characterized by a positive operator of trace one and, in turn, by the Fourier-Weyl transform of this generating operator. We demonstrate that informational completeness of the observable is a special case of a more general notion, namely the property that the span of translates of the generating operator is norm dense in the Schatten-p class. We show that the problem of characterisation of this property can be mapped completely to the corresponding problem for functions in classical harmonic analysis.

Effective dynamics of quantum dynamical semigroups – linear and non-linear
P. Lugiewicz

Abstract

The basic theorems about the destruction of quantum state-coherence in infinite dimensional systems will be discussed by the use of an algebraic approach. Examples of classical and quantum effective dynamics in the algebraic framework will be given. The notion of non-linear Markovian semi-group on C*-algebras will be proposed.

Decoherence induced by quantum measurements
R. Rebolledo

Abstract

For a number of physicists, *decoherence* consists of the loss of the so called coherences due to the coupled dynamics of an open system and its environment. In a seminal paper [4] Blanchard and Olkiewicz analyzed the *environmental decoherence* due to dissipation. Their approach was based on a decomposition of the algebra supporting the observables of the quantum system. This work influenced the characterization of the *decoherence-free* sub-algebra done in [11] and [12]. And has been extended to positive semigroups in [6]. So, the dynamics is represented here by a non-commutative semigroup and quantum environmental decoherence associated with dissipative phenomena.

The generator $\mathcal{L}$ of a quantum (or Markov) dynamical semigroup $\mathcal{T}$ is commonly supposed to be given at the outset. It is indeed often obtained by different limiting procedures leading to the so-called *master equations*, via adequate re-normalizations on time and space (see the book [1] which contains a systematic study of these techniques synthesized in the concept of *stochastic limit*).

The subject, from the physical point of view, is certainly much more complex than this version of decoherence. Several authors pointed out that a most careful analysis of involved time scales should be considered (see [19]). That is; decoherence should be for some researchers a phenomenon which precedes the derivation of the so-called Markov approximation of the dynamics.

Since the introduction of the concept by Zurek, and besides its connection with the foundations of quantum physics, decoherence has been frequently observed and studied from the experimental viewpoint. The interested reader is invited to follow the reports of the Haroche's group at the ENS in Paris [5], and that of the Wineland's group in Boulder [14]. Accordingly, a systematic insight on this phenomenon via quantum measurement theory is still an interesting open problem.

The conference will focus on the characterization of some mathematical problems connected with decoherence of quantum Markov semigroups induced by quantum measurements, illustrated via a number of examples inspired from physics.

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Renormalized Quantum Tomograhy
M. Sacchi

Abstract

We introduce a general framework to generate operator expansions for quantum tomography through the Kolmogorov construction. In fact, the usual theory of frames is suited to derive complete sets of observables and dual sets for obtain estimators just for bounded operators, whereas a unifying approach to classify operator expansions for unbounded operators is up to now unavailable. We show the role of null estimators in leading to many alternate expansions of the same operator over a quorum of observables, even making such expansions convergent for unbounded operators. As a byproduct, a number of new operator expansions has been found. Hopefully, our results will contribute to the solution of the problem of quantum tomography in infinite dimension, where a complete classification of convergent operator expansions over the quorum is still missing.

Ergodicity and decoherence for Generic Quantum Markov Semigroups
E. Sasso

Abstract

We study the relationships between ergodicity and environment induced decoherence for Quantum Markov Semigroups on a von Neumann algebra. We show that these properties are equivalent when the set of fixed points is an algebra containing the maximal subalgebra on which the semigroup is authomorphic. We apply these results to study the ergodicity and decoherence for generic quantum Markov semigroups: in particular, we highlight the strong relationships among these properties, the convergence in law of the classical Markov process associated to the diagonal part of the semigroup, and the structure of its communication classes.

Informationally complete Gaussian measurements
J. Shultz

Abstract

We prove necessary and sufficient conditions for the informational completeness of an arbitrary set of Gaussian observables, i.e., observables which give Gaussian outcome distributions for Gaussian states. In particular, we show that an informationally complete set either contains a single informationally complete observable, or includes infinitely many observables. We then generalize the treatment to the case where the measurement coupling is given by a general linear bosonic channel, and characterize informationally completeness for an arbitrary set of corresponding observables.

Normal completely positive maps on the space of quantum operations
A. Toigo

Abstract

We define a class of higher-order linear maps that transform quantum operations into quantum operations and satisfy suitable requirements of normality and complete positivity. For this class of maps we prove two dilation theorems that are the analogues of the Stinespring and Radon-Nikodym theorems for quantum operations. A structure theorem for probability measures with values in this class of higher-order maps is also derived.

Master equations from piecewise dynamics

B. Vacchini

Abstract

We consider how to construct classes of master equations beyond the Lindblad structure, still ensuring complete positivity of the dynamics. The result is obtained relying on a piecewise description of the time evolution and using as building blocks certain completely positive trace preserving maps whose action is distributed in time according to a general renewal process. We further study the relationship with previous results and the non-Markovianity of the obtained dynamics as quantified by the trace distance criterion.