DECOHERENCE FOR QUANTUM MARKOV SEMIGROUPS ON MATRIX SPACES

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Abstract

Decoherence is a phenomenon due to the interaction of the quantum system with its environment, and it has been accepted as the mechanism responsible for the appearance of classicality in quantum dynamics: see for example Zurek [7] and the monograph [5].
The most popular mathematical definition of decoherence was introduced by Blanchard and Olkiewicz in [1]: the key point is that, if decoherence takes place, the algebra describing the system can be decomposed in a maximal subalgebra $\mathcal{M}_1$ on which the evolution is reversible (the decoherence-free sector), and in a complementary subspace $\mathcal{M}_2$ on which the dynamics vanishes in time. Hence, after a sufficiently long time, the system behaves effectively like a closed system, and $\mathcal{M}_1$ is the effective algebra of observables after decoherence.
According to this definition, we describe a necessary and sufficient condition for decoherence of quantum Markov evolutions acting on matrix spaces. This condition is related to the spectral analysis of the generator $\mathcal{L}$ of the semigroup and is easily stated: the evolution displays decoherence if and only if the maximal algebra $\mathcal{M}_1$ on which the semigroup is $*$-automorphic contains all the eigenvalues of $\mathcal{L}$ associated with eigenvectors with null real part. Moreover, this condition is surely verified when the semigroup admits a faithful invariant state.

References

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