Analysis and detection of singularities using the continuous shearlet transform

The analysis and detection of singularities is a fundamental problem in mathematics and applications. In the study of the wave equation, for example, one is interested in the evolution of moving fronts, while, in computer vision and image processing, the detection and analysis of edges is an essential task for applications such as shape recognition or image enhancement and restoration. Multiscale methods and wavelets have been remarkably successful in this area, due to a combination of useful micro-analytical properties and fast discrete implementations. The continuous wavelet transform, in particular, has the ability to signal the location of the singularities of a function or a distribution through its asymptotic decay at fine scales. However, the classical wavelet approach is unable to provide additional information about the geometry of the set of singularities. This limitation can be overcome by considering one of the more sophisticated multiscale methods emerged during the last decade, such as the continuous shearlet transform. By combining the power of multiscale analysis with ability to handle anisotropic information very efficiently, the shearlet transform is able to provide a precise geometric characterization of singularities of function or distributions. In these lectures, I will first give an overview of the microlocal properties of wavelets and shearlets. Next, I will discuss a set of results illustrating the unique ability of the shearlet transform to provide a precise geometric characterization of singularities. In particular, I will show how to use this approach to identify the location and orientation of edge-type discontinuities. Finally, I will show the potential of these ideas in a range of applied problems including image denoising, edge detection and feature extraction.